Gargrave CE (VC) Primary School

Maths Calculation Policy

Progression in Calculations

Concrete Pictorial Abstract Addition

+ = signs and missing numbers

MENTAL

Adding 1 (eg. 7+1 and 1+7)

Doubles of numbers to 5

(eg 4+4) Adding 2 (eg 4+2 and 2+4)

Number bonds to 10 (eg 8+2 and 2 +8)

Alongside: Partitioning 2,3,4,5,6 and 10

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

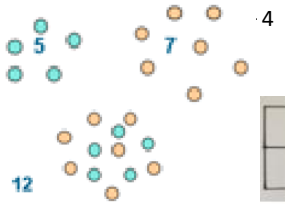
2 = 1+ 1

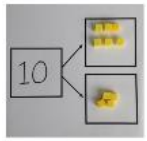
2 + 3 = 4 + 1

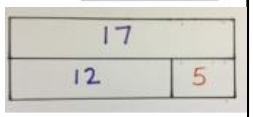
Missing numbers need to be placed in all possible places.

3 + 4 = □ □ = 3 + 4

4

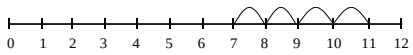






Counting and Combining sets of Objects

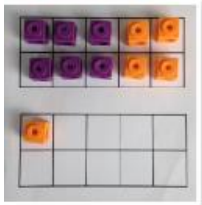
Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation)



Understanding of counting on with a numbertrack.

Understanding of counting on with a numberline (supported by models and images).

6+5



Start with the bigger number and partition the smaller number to make 10.

Concrete Pictorial Abstract Addition

Missing number problems e.g 14 + 5 = 10 + □ 32 + □ + □ = 100

35 = 1 + □ + 5

MENTAL

Adding 10, 0, doubles, near doubles, partitioning 7,8,9, bridging (8+4) compensating (9+4 = 10+4-1). Look for patterns and use partitioning and known additive facts. Partition into tens and ones then add and recombine/regroup/ exchanging.

It is valuable to use a range of representations (also see Y1).

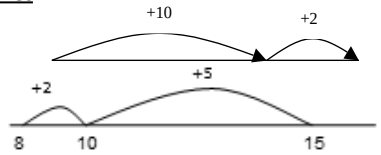
Continue to use numberlines to develop understanding of:

Counting on in tens and ones

23 + 12 = 23 + 10 + 2

= 33 + 2

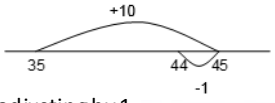
= 35



Partitioning and bridging tnrough 10.

The steps in addition often bridge through a multiple of 10 e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.

8 + 7 = 15



Adding 9 or 11 by adding 10 and adjusting by 1

e.g. Add 9 by adding 10 and adjusting by 1

35 + 9 = 44

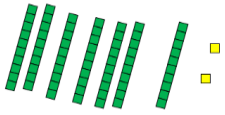
Towards a Written Method

Partitioning in different ways and recombine

47+25

47 + **25** = **60 + 12**

Leading to **72**



40 + 7

+ 20 + 5

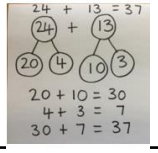
60 + 12 = 72

Expanded written method

40 + 7 + 20 + 5 =

40+20 + 7 + 5 =

60 + 12 = 72



Concrete Pictorial Abstract Addition

Missing number problems using a range of equations but with appropriate, larger numbers.

MENTAL

Mentally partition into hundreds, tens, ones then add, recombine/regroup with exchanging.

Partition into tens and ones

Partition both numbers and recombine.

Count on by partitioning the second number only e.g.

247 + 125 = 247 + 100 + 20 + 5

= 347 + 20 + 5

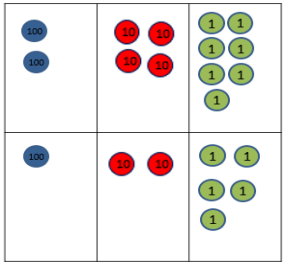
= 367 + 5

= 372

Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.

Towards a Written Method

Introduce expanded column addition modelled with place value counters (Dienes could be used for those who need a less abstract representation)



200 + 40 + 7

100 + 20 + 5

300 + 60 +12 = 372

247

+125

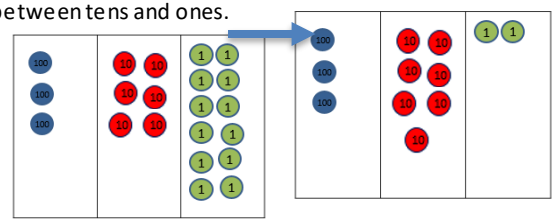
12

60

300

372

Leading to children understanding the exchange between tens and ones.



Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

247

+125

372

10

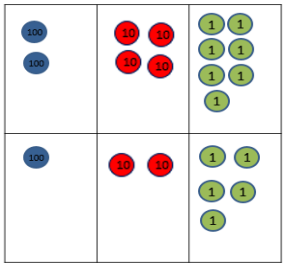
Concrete Pictorial Abstract Addition

Missing number/digit problems:

Mental methods Mentally partitioning then recombine/regroup including exchanging. Number lines and bar models should continue to be used to help with problem solving.

Written methods (progressing to 4-digits)

Expanded column addition modelled with place value counters, progressing to calculations with 4-digit numbers.

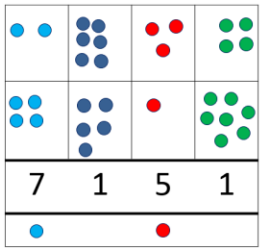


200 + 40 + 7
100 + 20 + 5
300 + 60 +12 = 372

247
+125
12
60
300
372

Compact written method

Extend to numbers with at least four digits.



2634
+4517
7151
1 1

Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty.

Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).

1
72.8
+ 54.6
127.4

Concrete Pictorial Abstract Addition

Missing number/digit problems:

Mental methods Mentally partitioning then recombine/regroup including exchanging. Number lines and bar models should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency

e.g. 12462 + 2300 = 14762

Written methods (progressing to more than 4-digits)

Progression should be when understanding of the expanded method is secure, children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.

1 1 1
172.83
+ 54.68
227.51

Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers.

Concrete Pictorial Abstract Addition

Missing number/digit problems:

Mental methods Mentally partitioning then recombine/regroup including exchanging. Number lines and bar models should continue to be used to help with problem solving.

Written methods

Progression to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured.

Continue calculating with decimals, including those with different numbers of decimal places

2	3	.	3	6	1
	9	.	0	8	0
5	9	.	7	7	0
+	1	.	3	0	0
9	3	.	5	1	1
2	1		2		

Problem Solving

Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding.

Concrete Pictorial Abstract Subtraction

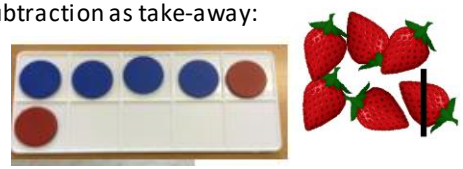
Missing number problems e.g. 7 = □ - 9; 20 - □ = 9; 15 – 9 = □; □ - □ = 11; 16 – 0 = □

MENTAL

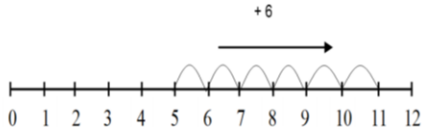
Adding 1, 2,doubling, bonds to 10, partitioning.

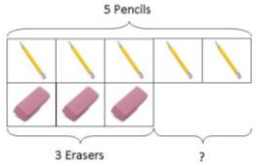
Use concrete objects and pictorial representations.

If appropriate, progress from using number lines with every number shown to number lines with significant numbers shown. Understand subtraction as take-away:



Understand subtraction as finding the difference:





The above model would be introduced with concrete objects which children can move (including cards with pictures) before progressing to pictorial representation.

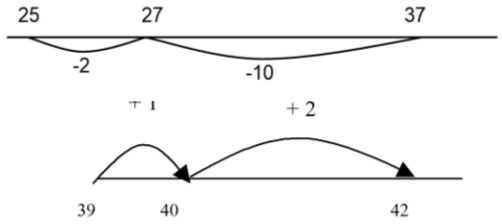
The use of other images is also valuable for modelling subtraction e.g. Numicon, bundles of straws, Dienes apparatus, multi-link cubes, bead strings

Concrete Pictorial Abstract Subtraction

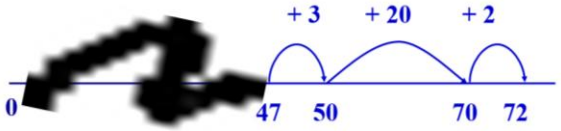
Missing number problems e.g. 52 – 8 = □; □ – 20 = 25; 22 = □ – 21; 6 + □ + 3 = 11

MENTAL

It is valuable to use a range of representations . Continue to use number lines to model take-away and difference. E.g.



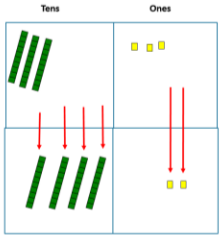
The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25.



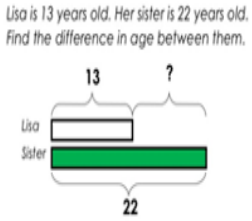
The bar model should continue to be used, as well as images in the context of **measures**.

Towards written methods

Recording addition and subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus. E.g. 75 – 42



70 5
-40 2
30 3



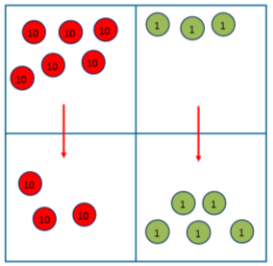
Concrete Pictorial Abstract Subtraction

Missing number problems e.g. □ = 43 – 27; 145 – □ = 138; 274 – 30 = □; 245 – □ = 195; 532 – 200 = □; 364 – 153 = □

Mental Number lines, bar model should continue to be used to help with problem solving Children should make choices about whether to use complementary addition or counting back, depending on the numbers involved.

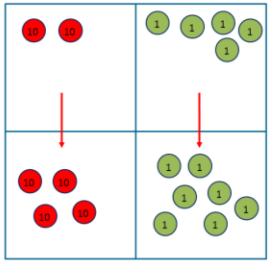
Written methods (progressing to 3-digits)

Introduce expanded column subtraction with no decomposition, modelled with place value counters (Dienes could be used for those who need a less abstract representation)



90 8
- 30 5
60 3

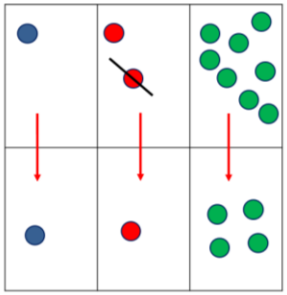
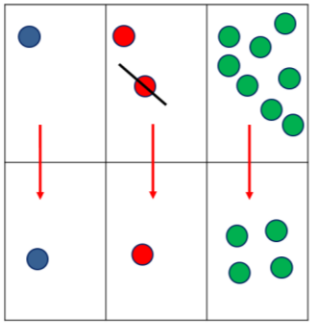
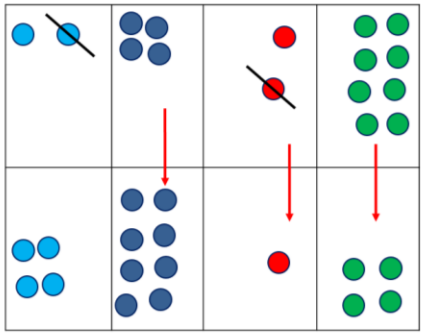
For some children this will lead to exchanging, modelled using place value counters (or Dienes).



60 ¹
70 2
- 40 7
20 5

A number line and expanded column method may be compared next to each other.

Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

Concrete Pictorial Abstract **Subtraction**	Concrete Pictorial Abstract **Subtraction**	Concrete Pictorial **Subtraction**
Missing number/digit problems: 456 + □ = 710; 1□7 + 6□ = 200; 60 + 99 + □ = 340; 200 – 90 – 80 = □; 225 - □ = 150; □ – 25 = 67; 3450 – 1000 = □; □ - 2000 = 900 **Mental** Apply what they know using inverse. Doubling, bonds, partitioning. Number lines and bar models should continue to be used to help with problem solving. **Written methods (progressing to 4-digits)** Expanded column subtraction with decomposition, modelled with place value counters, progressing to calculations with 4-digit numbers.  200 30 2 - 100 10 4 100 10 8 If understanding of the expanded method is secure, children will move on to the formal method of decomposition, which again can be initially modelled with place value counters.  232 - 114 118	Missing number/digit problems: 6.45 = 6 + 0.4 + □; 119 - □ = 86; 1 000 000 - □ = 999 000; 600 000 + □ + 1000 = 671 000; 12 462 – 2 300 = □ **Mental methods** Number lines and bar models should continue to be used to help with problem solving. **Written methods (progressing to more than 4-digits)** When understanding of the expanded method is secure, children will move on to the formal method of decomposition, which can be initially modelled with place value counters.  6232 - 4814 1418 Progress to calculating with decimals, including those with different numbers of decimal places.	Missing number/digit problems: □ and # each stand for a different number. # = 34. # + # = □ + □ + #. What is the value of □? What if # = 28? What if # = 21 10 000 000 = 9 000 100 + □ 7 – 2 x 3 = □; (7 – 2) x 3 = □; (□ - 2) x 3 = 15 **Mental methods** Number lines and bar models should continue to be used to help with problem solving. **Written methods** As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with decomposition to be secured. Teachers may also choose to introduce children to other efficient written layouts which help develop conceptual understanding. For example: 326 -148 -2 -20 200 178 Continue calculating with decimals, including those with different numbers of decimal places.

Concrete Pictorial Abstract Division

Children must have secure counting skills- being able to confidently count in 2s, 5s and 10s.
Children should be given opportunities to reason about what they notice in number patterns.

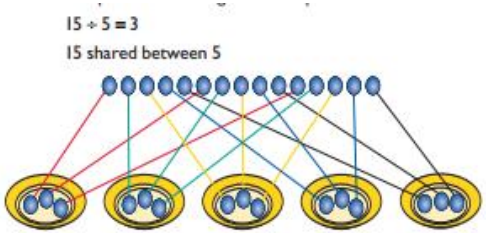
Mental methods

Applying inverse of year group multiplication facts

Group AND **share small quantities- understanding the difference between the two concepts.**

Sharing

Develops importance of one-to-one correspondence.

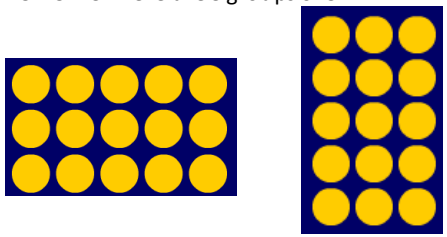


Children should be taught to share using concrete apparatus.

Grouping

Children should apply their counting skills to develop some understanding of grouping.

Use of arrays as a pictorial representation for division.
15 ÷ 3 = 5 There are 5 groups of 3.
15 ÷ 5 = 3 There are 3 groups of 5.



Children should be able to find ½ and ¼ and simple fractions of objects, numbers and quantities.

Concrete Pictorial Abstract Division

÷ = signs and missing numbers

6 ÷ 2 = □ □ = 6 ÷ 2
6 ÷ □ = 3 3 = 6 ÷ □
□ ÷ 2 = 3 3 = □ ÷ 2
□ ÷ ∇ = 3 3 = □ ÷ ∇

Mental methods

Applying inverse of year group multiplication facts.

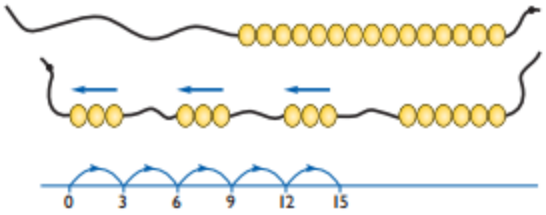
Know and understand sharing and grouping- introducing children to the ÷ sign.

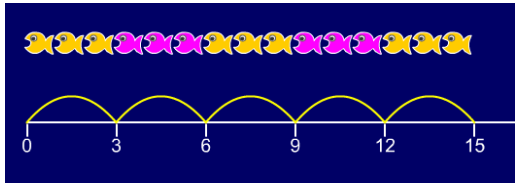
Children should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations.

Grouping using a numberline

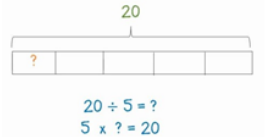
Group from zero in jumps of the divisor to find our 'how many groups of 3 are there in 15?'.

15 ÷ 3 = 5





Continue work on arrays. Support children to understand how multiplication and division are inverse. Look at an array– what do you see?



Concrete Pictorial Abstract Division

÷ = signs and missing numbers

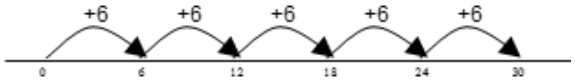
Continue using a range of equations as in year 2 but with appropriate numbers.

Mental methods

Applying inverse of year group multiplication facts.
Understanding repeated subtraction to find how many groups of a number are in the whole.

Grouping

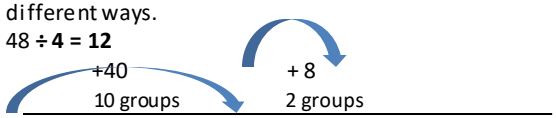
How many 6's are in 30?
30 ÷ 6 can be modelled as:



Becoming more efficient using a numberline

Children need to be able to partition the dividend in different ways.

48 ÷ 4 = 12



Remainders

49 ÷ 4 = 12 r1

10 groups 2 groups

Sharing – 49 shared between 4. How many left over?
Grouping – How many 4s make 49. How many are left over?

Place value counters can be used to support children apply their knowledge of grouping.
For example:
60 ÷ 10 = How many groups of 10 in 60?
600 ÷ 100 = How many groups of 100 in 600?

Concrete Pictorial Abstract Division

÷ = signs and missing numbers

Continue using a range of equations as in year 3 but with appropriate numbers.

Sharing, Grouping and using a number line

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. Children should progress in their use of written division calculations:

- Using tables facts with which they are fluent
- Experiencing a logical progression in the numbers they use, for example:

1. Dividend just over 10x the divisor, e.g. 84 ÷ 7
2. Dividend just over 10x the divisor when the divisor is a teen number, e.g. 173 ÷ 15 (learning sensible strategies for calculations such as 102 ÷ 17)
3. Dividend over 100x the divisor, e.g. 840 ÷ 7
4. Dividend over 20x the divisor, e.g. 168 ÷ 7

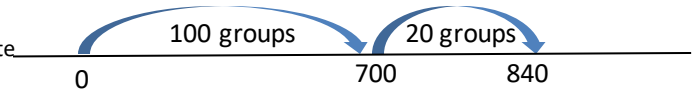
All of the above stages should include calculations with remainders as well as without.

Remainders should be interpreted according to the context. (i.e. rounded up or down to relate to the answer to the problem)

e.g. 840 ÷ 7 = 120

Jottings

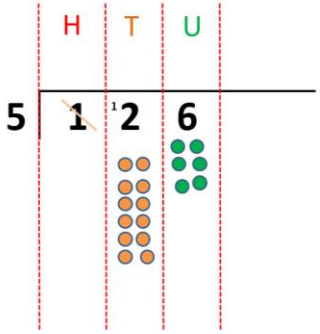
7 x 100 = 700
7 x 10 = 70
7 x 20 = 140



Formal Written Methods

Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines above)

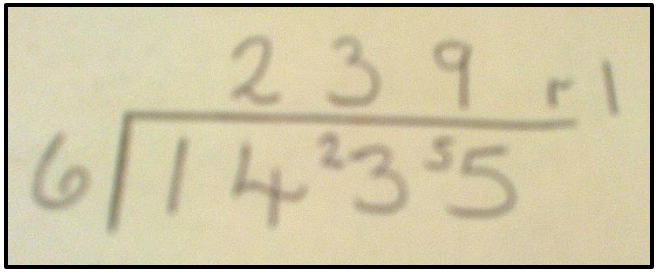
Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends. E.g. fig 1



Formal Written Methods

Continued as shown in Year 4, leading to the efficient use of a formal method. The language of grouping to be used.

E.g. 1435 ÷ 6



Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?)

Concrete Pictorial Abstract Division

÷ = signs and missing numbers

Continue using a range of equations but with appropriate numbers

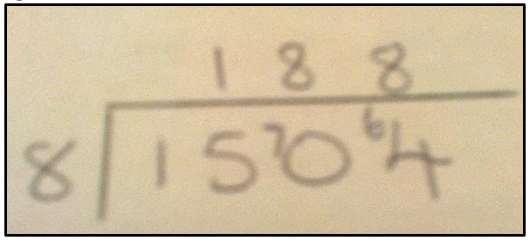
Sharing and Grouping and using a number line

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line as appropriate.

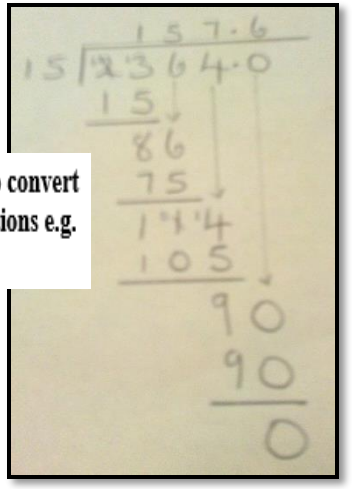
Quotients should be expressed as decimals and fractions

Formal Written Methods – long and short division

E.g. 1504 ÷ 8



E.g. 2364 ÷ 15



Challenge children to convert remainders into fractions e.g.
29 ÷ 4 = 7 ¼

29 ÷ 4 = 7 ¼
¼ = 0.25

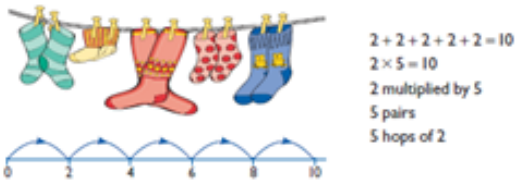
29 ÷ 4 = 7.25

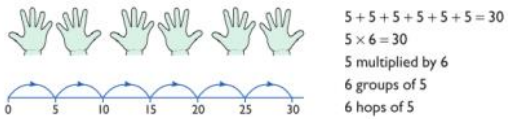
Concrete Pictorial Abstract Multiplication

Understand multiplication is related to doubling and combing groups of the same size (repeated addition)

Mental methods – count in equal groups

Washing line, and other practical resources for counting. Concrete objects. Numicon; bundles of straws, bead strings

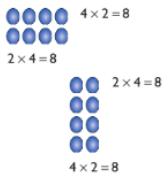


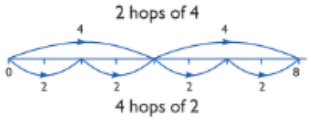


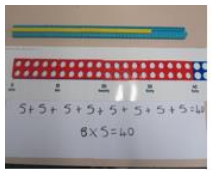
Problem solving with concrete objects (including money and measures

Use cuissenaire and bar method to develop the vocabulary relating to 'times' –
Pick up five, 4 times

Use arrays to understand multiplication can be done in any order (commutative)







Concrete Pictorial Abstract Multiplication

Expressing multiplication as a number sentence using x

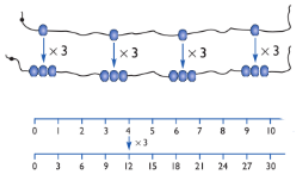
Using understanding of the inverse and practical resources to solve missing number problems.

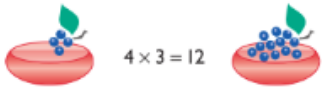
Mental methods – Multiplying numbers by 2, addition of 1,2,3 digit numbers. Counting forwards and backwards in multiples of numbers.

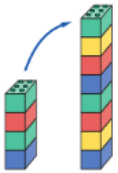
7 x 2 = □	□ = 2 x 7
7 x □ = 14	14 = □ x 7
□ x 2 = 14	14 = 2 x □
□ x ◯ = 14	14 = □ x ◯

Develop understanding of multiplication using array and number lines . Include multiplications not in the 2, 5 or 10 times tables.

Begin to develop understanding of multiplication as scaling (3





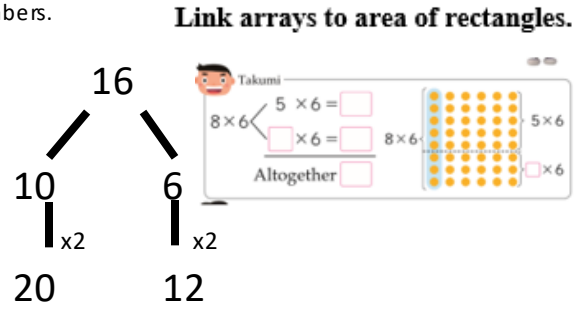


double 4 is 8
4 × 2 = 8

Doubling numbers up to 10 + 10
Link with understanding scaling
Using known doubles to work out double 2d numbers
(double 15 = double 10 + double 5)

Towards written methods

Use jottings to develop an understanding of doubling two digit numbers.



Concrete Pictorial Abstract Multiplication

Missing number problems

Continue with a range of equations but with appropriate numbers.

Mental methods

Doubling 2 digit numbers using partitioning

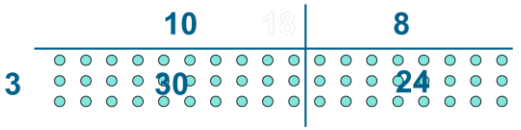
Relate repeated addition to multiplicative facts. Use reasoning strategies based on known facts eg 6x4 is 5x4 add one more group of 4. Instant recall. Use knowledge of multiplication facts to increase number sizes.

Demonstrating multiplication on a number line – jumping in larger groups of amounts

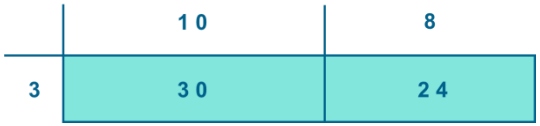
13 x 4 = 10 groups 4 = 3 groups of 4

Written methods

Developing written methods using understanding of visual images

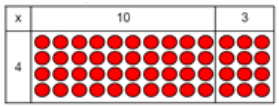


Develop onto the grid method



Give children opportunities for children to explore this and deepen understanding using Dienes apparatus and place value counters

18 x 3 =



4 rows

X	10	8
3	30	24

30 + 24 = 54

Concrete Pictorial Abstract Multiplication

Continue with a range of equations but with appropriate numbers. Also include equations with missing digits

□2 x 5 = 160

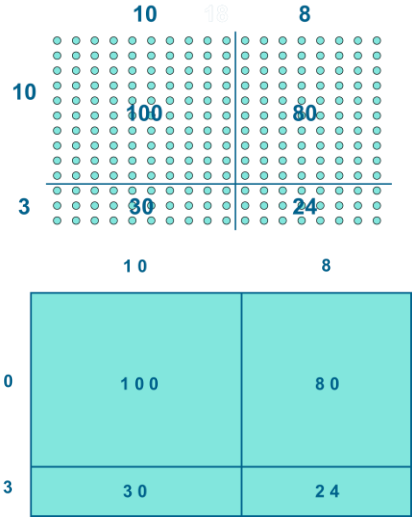
Mental methods

Counting in multiples of 6, 7, 9, 25 and 1000, and steps of 1/100. Use knowledge of multiplication facts to increase number size.

Solving practical problems where children need to scale up. Relate to known number facts. (e.g. how tall would a 25cm sunflower be if it grew 6 times taller?)

Written methods (progressing to 3d x 2d)

Children to embed and deepen their understanding of the grid method to multiply up 2d x 2d. Ensure this is still linked back to their understanding of arrays and place value counters.



Concrete Pictorial Abstract Multiplication

Continue with a range of equations but with appropriate numbers. Also include equations with missing digits

Mental methods

X by 10, 100, 1000 using moving digits ITP. Use knowledge of multiplication facts to increase number size.

Use practical resources and jottings to explore equivalent statements (e.g. 4 x 35 = 2 x 2 x 35)

Recall of prime numbers up 19 and identify prime numbers up to 100 (with reasoning)

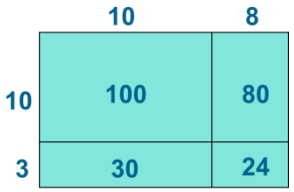
Solving practical problems where children need to scale up. Relate to known number facts.

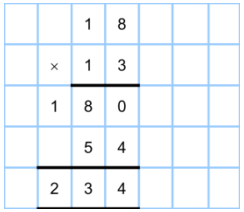
Identify factor pairs for numbers

Written methods (progressing to 4d x 2d)

Long multiplication using place value counters

Children to explore how the grid method supports an understanding of long multiplication (for 2d x 2d)





Concrete Pictorial Abstract Multiplication

Continue with a range of equations but with appropriate numbers. Also include equations with missing digits

Mental methods

Identifying common factors and multiples of given numbers. Solving practical problems where children need to scale up. Relate to known number facts. Instant recall for efficiency.

Written methods

Continue to refine and deepen understanding of written methods including fluency for using long multiplication